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RESEARCH ON THE DEFORMATIONAL AND VIBRATIONAL RESPONSE OF SUBMARINE-TYPE HULL STRUCTURES TO UNDERWATER BLAST LOADING--1965-1970

by

Joseph Kempner

Polytechnic Institute of Brooklyn
Department of
Aerospace Engineering and Applied Mechanics

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CONTENTS

*Numbers in brackets refer to publications

I. INTRODUCTION

The studies briefly summarized in this report stem from investigations of blast effects on submarine structures, and from related static and dynamic analyses of structural components. The work was performed or published during the past five years by the Department of Aerospace Engineering and Applied Mechanics of the Polytechnic Institute of Brooklyn on Contract No. Nonr 839(40), Project No. NR-067-427.

As indicated by the table of contents, the work covered shell-fluid interaction problems, dynamics of layered media, three-dimensional elasticity analyses, and orthotropic plates and shells. The summary presents brief abstracts of the material presented in twenty-three distinct reports and publications. Additionally, thirteen theses and dissertations are listed by title only.

The writer would like to take this opportunity to acknowledge the financial assistance and encouragement provided by the Office of Naval Research throughout the course of the studies summarized herein.

2. SHELL-FLUID INTERACTION PROBLEMS

2.1 Herman, Harry and Klosner, J. M.: Transient Response of a Periodically Supported Cylindrical Shell Immersed in a Fluid Medium. Jour. Appl. Mech., Vol. 32, Series E, No. 3, September 1965, pp. 562-568; presented at Western Conf. of Appl. Mech. Div., ASME, Aug. 1965; formerly, PIBAL Rep. No. 690.

The dynamic response of a periodically simply-supported, infinitely long, circular, cylindrical shell to a pressure suddenly applied through the surrounding acoustic medium is investigated. The incident particle velocity is zero, and the pressure is assumed to have a harmonic spatial variation parallel to the shell axis. The exact solution is obtained by use of a Fourier integral transform, and the resulting inversion integral is evaluated by numerical and asymptotic integration. Two solutions to the same problem are obtained by using a plane and cylindrical wave approximation for the radiated field. The range of their applicability is investigated. For a steel shell in water it is found that, when the supports are placed three shell diameters apart, the use of the cylindrical wave approximation results in a 5-percent underestimation of the maximum deflection, while when the supports are placed one sixth of a shell diameter apart, the approximations are invalid.

2.2 Klosner, J. M. and Berglund, J.: A Steady-State Formulation for the Transient Response of a Reinforced Cylindrical Shell Immersed in a Fluid Medium. PIBAL Rep. No. 932, June 1967; also, Jour. of Appl. Mech., Vol. 34, No. 2, June 1967, pp. 487-490.

The transient problem previously solved by Herman and Klosner by using a generalized Fourier transform to suppress the time coordinate is re-solved by using a method that reduces it to an equivalent steady-state problem.

Results show that the method is indeed expedient and thus can be applied to other problems which would otherwise be untractable.

2.3 Berglund, Jerry W. and Klosner, Jerome M.: Interaction of a Ring-Reinforced Shell and a Fluid Medium. PIBAL Rep. No. 935, June 1967; also, Jour. of Appl. Mech., Vol. 35, No. 1, March 1968, pp. 139-147, presented at the Nat'l. Appl. Mech. Conf., Brown Univ., June 12-14, 1968.

This work is concerned with the transient dynamic response of a periodically ring-reinforced, infinitely long, circular cylindrical shell to a uniform pressure suddenly applied through the surrounding acoustic medium. The incident particle velocity is zero and the rings are assumed to be slightly flexible.

A classical theory of the Donnell type is used to analyze the shell while the fluid is described by the linear acoustic field equation.

The solution is obtained by assuming a power series expansion in the ring stiffness parameter and utilizing a technique which reduces the transient dynamic problem to an equivalent steady-state formulation.

Numerical results are presented for a steel shell immersed in salt water for different ring spacings.

For the case of rigid rings, a cylindrical and plane wave approximation was also used to represent the fluid field. It is shown that the cylindrical wave approximation yields reasonably accurate results.

Flexible ring results, although limited, indicate that undamped or non-radiating components of the shell vibration are activated.

2.4 Fang, Tsun C. and Klosner, Jerome M.: Expanding Axial Wave on a Submerged Cylindrical Shell. PIBAL Rep. No. 69-2, January 1969; accepted for publication Quart. Appl. Mech., presented at 6th U. S. Nat'l. Congress of Appl. Mech., Harvard Univ., June 15-19, 1970.

A double transform method is used to determine the response of a submerged, infinitely long, circular, cylindrical shell to a plane acoustic wave which acts initially at an isolated cross section, and then proceeds to propagate along the axis of the cylinder, symmetrically with respect to that cross section.

The application of the Saddle Point Method to the inversion integral results in an asymptotic solution, valid for long time. Comparisons are made with the analogous steady-state solution obtained by using piston theory to describe the effects of the fluid on the shell.

2.5 Klosner, Jerome M.: Inadequacies of Acoustic Approximations in Fluid-Shell Interactions. PIBAL Rep. No. 69-11, April 1969; also, Jour. Engrg. Mechs. Div. ASCE, Vol. 96, No. EM 2, April 1969, pp. 143-159, presented at Louisville Conf. (ASCE) Annual Mtg., April 14-18, 1969.

The results of several investigations of the submerged circular cylindrical shell as well as some new results for the spherical shell are discussed, so as to shed additional light on the limitations of the plane (piston theory) and cylindrical wave approximation.

It should be noted that when non-radiating components of the shell-fluid response are significant, then the acoustic approximations inadequately predict the interaction phenomenon. Such components are significant when the motion of the structure induces closely-spaced (when compared to the fluid wavelength) adjacent compressed and rarified regions in the fluid. This is accompanied by an axial flow from the adjacent compressed and rarified regions. At high frequencies (short fluid wavelengths) this does not occur since sufficient time is not available.

2.6 Lou, Yung-Kia and Klosner, Jerome M.: Transient Response of a Ring-Stiffened Spherical Shell Immersed in an Acoustic Medium. PIBAL Rep. No. 69-12, April 1969; accepted for publication Jour. Appl. Mechs.

The transient response of a ring-stiffened spherical shell to a sudden pressure increase in the surrounding acoustic medium is investigated. A modal expansion approach is used to analyze the shell, while the acoustic field equation is solved by invoking the Helmholtz integral. Coupling of the two fields occurs through the enforcement of continuity of the velocity components at the shell-fluid interface.

Two solutions to the same problem are obtained by using plane and cylindrical wave approximations of the acoustic field. These approximate solutions fail to predict the transient behavior for the shell configuration analyzed.

The result of this study indicates that a large dynamic factor must be assumed in the design of submerged stiffened spherical shell structures, if explosive loads are likely to be encountered.

3. DYNAMICS OF LAYERED MEDIA

3.1 Armenàkas, Anthony E.: Torsional Waves in Composite Rods. Jour. Acous. Soc. Am., Vol. 38, No. 3, September 1965, pp. 439-446; formerly, PIBAL Rep. No. 699.

The propagation of torsional waves in composite, infinitely long, tractionfree, circular, cylindrical rods is investigated on the basis of the three-dimensional linear theory of elasticity. The composite rods consists of an internal circular rod made of one material, bounded by and bonded to a circular casing made of another material. The effect of the variation of the geometric and physical parameters on the frequency of the torsional motion is discussed. It is shown that, in general, for composite rods, the first torsional mode (no nodes along the radius) may not exist uncoupled.

3.2 Armenàkas, Anthony E.: Propagation of Harmonic Waves in Composite Circular Cylindrical Shells, Part I--Theoretical Investigation. PIBAL Rep. No. 924, May 1966; also, AIAA Jour., Vol. 5, No. 4, April 1967, pp. 740-744.

In this investigation, the propagation of harmonic waves of an arbitrary number of circumferential nodes travelling in composite, traction-free, circular cylindrical shells, is studied on the basis of the linear, three-dimensional theory of elasticity. The composite shells consist of a circular inner layer composed of one material, bounded by and bonded to a circular outer casing made of another material.

3.3 Armenàkas, Anthony E.: Propagation of Harmonic Waves in Composite Circular Cylindrical Rods. PIBAL Rep. No. 69-4, February 1969; also, Jour. Acous. Soc. of Am., Vol. 47, No. 3 (Part 2), March 1970, pp. 822-837.

In this investigation the general frequency equation for harmonic waves having an arbitrary number of circumferential nodes, traveling in composite, traction-free, circular cylindrical rods is established on the basis of the linear three-dimensional theory of elasticity. The composite rods consist of a circular core made of one material, bounded by and bonded to a circular casing of another material. Simpler degenerate cases of the frequency equation are reduced and discussed.

A numerical evaluation of the frequency equation is presented. The results are obtained by programming an iteration procedure on an IBM 7094 digital computer.

The effect of the variation of the physical and geometric parameters of the rod on the frequencies and mode shapes of the first few modes is illustrated and discussed. Moreover, the feasibility of utilizing composite rods as delay media in guided wave ultrasonic delay lines is considered briefly.

3.4 Armenàkas, Anthony E. and Keck, Henry E.: Harmonic Non-Axisymmetric Waves with Short Wavelengths Propagating in Composite Rods. PIBAL Rep. No. 69-34, August 1969; accepted for publication Jour. of Acous. Soc. of Am.

The asymptotic phase velocities at very large wave numbers of non-axisymmetric waves, traveling in composite rods are established analytically. Frequency spectra and mode shapes of composite rods are presented and compared with those of simple (made of one material) rods.

3.5 Chen Youl-Nan; Ranlet David and Kempner, Joseph: Free Vibrations of Free-Free Web-Stiffened Sandwich Beams. PIBAL Rep. No. 69-35, September 1969.

Presented herein is an analysis of the free vibrations of free-free, web-stiffened sandwich beams. The mathematical model formulated includes the effect of translatory and rotatory inertia in each layer of the sandwich, and assumes that the faces may be treated as thin layers in which classical beam theory applies. Shear deformations, however, are permitted in the core, which is treated as a layer of orthotropic material comprised of a series of webs considered as discrete elements, except for the inclusion of an average secondary shear modulus induced by the bending of the faces and the webs. Equations are also developed for the free vibrations of the corresponding homogeneously cored sandwich beams, in order to investigate the effect of smearing-out or averaging of a given web-stiffened core.

A Galerkin-type procedure is employed to determine the natural frequencies from a variational functional generated by means of Hamilton's principle. For free-free, web-stiffened sandwich beams of constant weight, the results show that smearing-out the web-stiffened core introduces significant error into the natural frequencies only when the mass in the core layer is distributed among a small number of webs. Moreover, it is demonstrated that, in general, the lower modes of a free-free, web-stiffened sandwich beam have higher frequencies than the corresponding modes of a free-free homogeneous beam having a rectangular cross section and the same length, width and weight. For the higher modes, the opposite is indicated.

3.6 Armenakas, Anthony E.: Propagation of Harmonic Waves in Composite Circular Cylindrical Shells. Part II: Numerical Analysis. PIBAL Rep. No. 69-44, October 1969; accepted for publication AIAA Jour.

The frequency equation for harmonic waves with an arbitrary number of circumferential nodes traveling in composite traction-free circular cylindrical shells established in the first part of this investigation has been programmed for numerical evaluation on an IBM 7094 digital computer. The numerical results obtained are employed in evaluating the effect of the changes of the shell parameters on the frequency and shape of the first few modes of wave propagation. Moreover, the asymptotic limits of the phase velocities for waves having short axial wavelengths are established analytically and verified by the numerical results.

3.7 Armenakas, Anthony E. and Keck, Henry E.: Wave Propagation in Three-Layered Plates--Application to Ultrasonic Delay Lines. PIBAL Rep. No. 70-6, February 1970.

The propagation of trains of straight crested harmonic waves in infinitely long isotropic, traction free, three-layered plates is investigated on the basis of the three-dimensional linear theory of elasticity. It is established, that as in the case of waves traveling in homogeneous plates the face shear motion and the longitudinal-flexural motion exist uncoupled. However, independent symmetric and anti-symmetric modes exist only in the case of three-layered symmetric plates; that is, plates having identical outer layers.

The frequency equations have been programmed on an IBM 7044/7094 DCS computer and evaluated numerically.

The possibility is discussed of utilizing thin three-layered symmetric strips as delay media in guided wave ultrasonic delay lines having a linear delay versus frequency relation.

3.8 Reitz, Edward S. and Armenakas, Anthony E.: Propagation of Harmonic Waves in Orthotropic Circular Cylindrical Shells. Part I--Theoretical Investigation. PIBAL Rep. No. 70-18, March 1970.

In this investigation the general frequency equation for trains of harmonic waves having an arbitrary number of circumferential nodes, traveling in orthotropic circular cylindrical shells is established on the basis of the three-dimensional linear theory of elasticity, by expanding the displacement components in power series of the radial coordinate.

Simpler forms of the frequency equation for axisymmetric non-torsional and torsional motion and for longitudinal-shear and plane strain motion are established and discussed.

4. THREE-DIMENSIONAL ELASTICITY ANALYSES

4.1 Klosner, Jerome M. and Levine, Howard S.: Further Comparisons of Elasticity and Shell Theory Solutions. AIAA Jour., Vol. 4, No. 3, March 1966, pp. 467-480, presented at Annual AIAA Mtg., January 1965; formerly, PIBAL Rep. No. 689.

The problem of an infinite circular cylindrical shell subjected to periodically spaced band loads is investigated using three-dimensional elasticity theory, the classical shell theories of Timoshenko (or Donnell) and Flügge, two shear deformation shell theories of the Timoshenko and Flügge type, and a higher-order shell theory developed by Reissner and Naghdi. A detailed comparison of the resulting stresses and displacements with the exact elasticity solution was carried out for ratios of inner to outer shell radius varying from 0.7 to 0.93, and for a ratio of distance between band loads to outer shell diameter equal to 0.2. The ratio of band width to distance between band loads was chosen to be 0.2, whereas the Poisson's ratio used was 0.3. It has been shown that when discontinuous transverse normal loads are effective over the length of a shell, as they are in the short shells considered here, one must not assume a priori that a higher-order shell theory will yield the most accurate results.

4.2 Levine, Howard S and Klosner, Jerome M.: Transversally Isotropic Cylinders under Band Loads. PIBAL Rep. No. 908, January 1966; also, Jour. Engrg. Mechs. Div., ASCE, Vol. 93, No. EM 3, June 1967, pp. 157-174, presented at ASCE Structural Engrg. Conf., Jan. 1966.

The problem of a transversally isotropic circular cylindrical shell subjected to periodically spaced, axisymmetric band loads is investigated. The three-dimensional elasticity solution is obtained and stresses and displacements are evaluated for values of the modulus ratio (E_r/E_z) equal to two, five and ten. Also presented are expressions for the stresses and displacements obtained by use of a thin orthotropic shell theory. The results are compared to determine the effects of a change in modulus ratio on the stresses and displacements of the cylinder and the accuracy of the shell theory.

A detailed analysis of the data is carried out for a ratio of inner to outer shell radii equal to 0.70, a ratio of distance between band loads to outer shell diameter equal to 0.2, and a ratio of band width to distance between band loads of 0.2.

4.3 Armenàkas, Anthony E.: On the Accuracy of Some Dynamic Shell Theories. PIBAL Rep. No. 910, February 1966; also, Jour. Engrg. Mechs. Div., ASCE, Vol. 93, October 1967, pp. 95-109.

The error in the frequency and mode shape obtained on the basis of the Flügge bending theory and a Donnell-type dynamic theory is established as a function of the shell parameters for modes having a small number of circumferential waves. Moreover, the effect on the frequency of the axial and tangential inertia is evaluated.

Finally, simple but accurate expressions for the frequency of the first mode are presented and discussed.

4.4 Levine, Howard S. and Klosner, Jerome M.: Axisymmetric Elasticity Solutions of Spherical Shell Segments. PIBAL Rep. No. 68-12, May 1968; accepted for publication Jour. of Appl. Mechs.

An exact series solution for the stresses and displacements of a spherical segment subjected to arbitrary axisymmetric surface tractions and edge boundary conditions is presented. The general solution for the axisymmetric case has been obtained by utilizing two sets of functions, namely, the Lur'e homogeneous functions and the full sphere functions used by Sternberg, Eubanks and Sadowsky. In particular, solutions to the following problems have been obtained: (a) the spherical segment with a stress free edge subjected to a centrifugal force field; (b) the spherical segment subjected to an external pressure varying as $\cos^{2N}\theta$ supported on a rigid surface with no shear resistance; (c) the hemisphere having zero traction on its spherical surfaces subjected to edge shear stresses.

The results are presented in graphic form, which demonstrate the boundary layer effect. Heretofore, solutions to these types of problems have been obtained by using shell theory approximations.

4.5 Franklin, Howard N. and Klosner, Jerome M.: Asymptotic Solution of a Thick Spherical Shell with Circular Holes. PIBAL Rep. No. 70-15, April 1970.

A singular perturbation analysis is developed for a spherical shell, containing two diametrically opposite holes, subjected to an axi-symmetric external pressure.

The asymptotic formulation decomposes the shell into three sub-regions, an interior region, a wide boundary layer, and a narrow boundary layer. The subregional solutions are matched and a uniformly valid solution is obtained.

Stress concentration factors, radial normal and shearing stresses, and displacements are presented for a hole subtended by a half angle of one-tenth of a radian ($\varphi_o = 0.1$). It is shown that the narrow boundary layer region is of the order of the shell thickness, while the wide boundary layer effects may be neglected at distances greater than $2\sqrt{Rh}$ from the hole.

5. ORTHOTROPIC PLATES AND SHELLS

5.1 Venkatraman, B. and Patel, Sharad A.: Dynamic Response of Orthotropic Plastic Cylindrical Shells under Radial Pressures. PIBAL Rep. No. 914, March 1966; also, Jour. of Franklin Inst., Vol. 282, No. 3, September 1966, pp. 171-178.

The dynamic response of orthotropic circular cylindrical shells subjected to exponentially decaying uniform radial pressures have been considered in the present paper. The initial values of the pressures applied are slightly larger than the corresponding static collapse pressures of the shells obtained on the basis of Tresca's yield criterion. The maximum kinematically admissible deflections for a cantilever shell as well as a shell clamped at both ends are presented for various values of the shell parameter.

5.2 Patel, Sharad A. and Broth, Franklin J.: Axisymmetric Buckling of Orthotropic Circular Plates with Variable Thickness. PIBAL Rep. No. 920, June 1966; also, Jour. of Royal Aero. Soc., Vol. 71, March 1967, pp. 218-223.

This report is concerned with the axisymmetric buckling of an orthotropic circular plate having variable thickness. The thickness varies in the radial direction and the plate is subjected to a uniform edge compression. The governing equation for the stress distribution due to this edge compression is derived and solved in terms of a power series in the radial coordinate. With the use of this stress distribution, the equation governing the buckling of the plate is derived. It is solved by Galerkin's method for a clamped plate as well as for a simply-supported plate. Values of the buckling loads thus obtained for a uniform thickness plate agree well with existing results.

5.3 Salzman, Alan P. and Patel, Sharad A.: Natural Frequencies of Orthotropic Circular Plates of Variable Thickness. PIBAL Rep. No. 68-8, April 1968; also, AIAA Jour., Vol. 6, No. 8, August 1968, pp. 1625-1626.

The present investigation is concerned with the natural frequencies of orthotropic circular plates of variable thickness. In particular, a thickness variation of the form $h = h_o(1 - HR^n)$ has been selected. The derivation of the differential equation governing the motion of the plate is based on the classical formulation of the theory of plates. The solution of this equation for the axisymmetric case is obtained by an application of the method of Frobenius. Characteristic equations for the natural frequencies of clamped and simply-supported plates are derived and numerical results are presented for several plates of various shapes.

5.4 Salzman, Alan P. and Patel, Sharad A.: Asymmetric Natural Vibrations of Orthotropic Circular Plates of Variable Thickness. PIBAL Rep. No. 69-6, March 1969.

Natural frequencies corresponding to asymmetric modes of vibration of orthotropic circular plates with variable thickness are investigated in the present report. The thickness variation chosen is of the form $h = h_o(1 - HR^n)$. The governing differential equation is solved by the method of separation of variables together with Frobenius' method. The characteristic equations for the natural frequencies of clamped and simply-supported plates are derived. Numerical values of frequencies corresponding to several modes of vibrations are presented for various particular plate shapes.

6. THESES AND DISSERTATIONS

6.1 Broth, Franklin J.: Axisymmetric Buckling of Orthotropic Circular Plates with Variable Thickness. M.S. (Applied Mechanics), June 1966.

6.2 Shulman, Michael.: Effects of Normal Stress and Transverse Shear Deformation on a Circular Cylindrical Shell Subjected to Hydrostatic Pressure. M.S. (Applied Mechanics), June 1966.

6.3 Berglund, Jerry W.: Interaction of a Ring-Reinforced Shell and a Fluid Medium. Ph.D (Applied Mechanics), June 1967.

6.4 Levine, Howard S.: Axisymmetric Elasticity Solutions of Spherical Shell Segments. Ph.D (Applied Mechanics), June 1968.

6.5 Salzman, Alan P.: Natural Frequencies of Orthotropic Circular Plates of Variable Thickness. M.S. (Applied Mechanics), June 1968.

6.6 Martin, James.: Dispersion of Flexural Waves in Composite Elastic Circular Rods. M.S. (Applied Mechanics), June 1969.

6.7 Lou, Yung Kia.: Transient Response of a Ring-Stiffened Spherical Shell Immersed in an Acoustic Medium. Ph.D (Applied Mechanics), June 1969.

6.8 Fang, Tsün C.: Expanding Axial Wave on a Submerged Cylindrical Shell. Ph.D (Applied Mechanics), June 1969.

6.9 Golub, Eugene B.: Shell Theory for Thick Circular Cylinders. Ph.D (Applied Mechanics), June 1969.

6.10 Ranlet, David.: Free Vibrations of Circular Cylindrical Web-Stiffened Sandwich Shells. Ph.D (Applied Mechanics), June 1969.

6.11 Pulos, John G.: A Unified Account of Analytical Methods for Upper and Lower Bounds to Eigen-values of Beam and Column Problems. Ph.D (Applied Mechanics), June 1969.

6.12 Bartenhagen, John S. Jr.: Stresses and Dispalcements in Thick Beams. M.S. (Applied Mechanics), June 1970.

6.13 Franklin, Howard N.: Asympotic Solution of a Thick Spherical Shell with Circular Holes. Ph.D (Applied Mechanics), June 1970.